

An Efficient Iterative Double Auction for Energy Trading in Microgrids

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Abstract—This paper proposes a double auction mechanism for energy trade between buying and selling agents. The framework is general enough, requiring neither the agents' preferences nor the energy pricing to be fixed values across the spatially distributed agents. A microgrid controller implements a distributed algorithm to maximize individual participating agents' utilities as well as the social welfare. This is accomplished by the controller in an iterative manner, such that the need to obtain private information pertaining to individual agents' preferences is obviated. Simulation results with a set of seven buyers and an equal number of sellers indicate that the proposed iterative double auction can establish social welfare maximization, requiring only a reasonable amount of computational overhead.

Keywords—microgrid; agents; trading; auction; bid; social welfare

I. INTRODUCTION

The future energy grid is evolving into a complex cyber physical network encompassing advanced bi-directional energy flow, communications, control, and energy trade [1]. Microgrids, being a key element of the future smart grid on the distribution side, are receiving much research attention. This may be attributed to inherent features such as environmental friendliness, reliability, self-sustainability and social benefits [2–4]. Consumers (buyers) and producers (sellers) of energy, which are collectively referred to as agents throughout this paper, along with the utility provider – a central control unit who owns the distribution network in the microgrid (hereafter referred to as the controller), constitute an energy market with agents, rendering the possibility of online energy trading during outages. Various trading algorithms that maximize the overall social welfare, i.e. the sum of the utilities of all agents, have been proposed. For example, Nash equilibria under several trading scenarios have been explored in [5–9, 20]. The application of non-cooperative games and coalition formation among trading agents has been explored in [5, 6]. Non-cooperative game theory and a double auction mechanism to determine the market price in order to maximize sellers' utilities has been reported in [10]. A similar multi-layer double auction scheme which deals with information flow between broker and selling agents, where the sellers determine their selling quantity in order to maximize their utility is studied in [21]. Unfortunately in [10] and [21], buyers' utilities from the trade, has not been considered. Additionally, key aspects of energy trading, such as maintaining agents' privacies within a

distributed agent framework, have been consistently ignored in these studies. In our approach, the network parameters are unknown and the broker has to solve a different optimization problem to elicit the hidden information and specific pricing rules and induce agents to bid in such a way that solving this different optimization problem will eventually solve the actual social maximization problem. Our approach specifically deals with privacy preservation whereas the previous methods fail to do so. Lastly, an underlying assumption in earlier research is that the price of each unit of energy is fixed across all agents.

In this paper, a more realistic energy trading mechanism that addresses the aforementioned issues, has been introduced. Before the actual energy trade takes place, an auction is held between all the agents, with the controller acting as the auctioneer. Due to the presence of both buyers and sellers, the proposed auction is cast within the framework of a double auction [12]. The auction process aims to facilitate energy trading between the buyers and sellers in such a manner that the overall utilities of both are maximized.

In order to account for geographically distributed locations of the buying and selling agents of a microgrid, the model considers asymmetric demand and supply. More specifically, instead of using a scalar term for each buying agent's demand or each selling agent's supply, each such agent incorporates a demand or supply vector. Furthermore, longer geographic distances make it necessary to consider transmission losses as well. In the present scheme, these losses are incorporated as subtractive terms within buyer's utility. Hence a buyer may prefer to acquire energy from sellers in its vicinity than those that are located farther away.

The finite bounds on the maximum energy that each seller can deliver and each buyer can consume are incorporated as hard constraints in the present formulation. Neither the total energy that each seller delivers during the trade, nor the total energy that a buyer consumes can exceed a threshold, which is different for each agent.

These maximum limits are automatically addressed by the controller during the ensuing auction, whose objective is to maximize the overall social welfare of all agents participating in the trade. This social welfare is merely the sum of the nonlinear utilities of the agents. Unfortunately, a realistic setting for energy trading mandates that these buyers' and sellers' utilities be treated as private information that is

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accessible neither to other agents nor the controller. To extract this private information of the sellers and buyers – assumed to be selfish as well as non-cooperative, an incentive compatible auction mechanism has been proposed (c.f. [11]).

Agents express their individual preferences indirectly by placing bids to the controller. The bid that each agent places, is dependent on its utility; furthermore no agent can expect to improve its utility by deviating from the bid's true value. It must be noted that in keeping with the general approach of maintaining geographic asymmetry, the bids have also been vectorized.

As a first step of the proposed auction, the buying and selling agents submit their initial bid vectors to the controller, designed to maximize the agents' utilities. It shall be noted that the demand and supply vectors of the other agents, their utilities, as well as the upper bounds, are not factored into these bids yet. Thus the auction must take place in an iterative manner. To the best of the authors' knowledge, such iterative double auctions have not been reported in the existing literature on energy grids. However, a parallel situation arises in the context of wireless communications, where auction mechanisms have been studied extensively for almost a decade. For instance, in a recent study, the problem of distributed data offloading between transmitters serving as agents, has been investigated by means of an iterative double auction [14, 15]. The scheme proposed in this research is adapted from that approach, with relevant changes that make the Iterative Double Auction (IDA) applicable to microgrid energy trading.

The remainder of this paper is organized in the following manner: Section II introduces the mathematical framework within which the controller as well as the participating agents of the microgrid have been modeled. Section III formulates the problem in terms of an iterative double auction while section IV details the algorithmic implementation of the auction. Simulation results are discussed at length in section V. Conclusion of this effort and suggested future research is outlined in the last section i.e. section VI.

II. FRAMEWORK

A. Problem description

Consider a microgrid with N agents as shown in Fig. 1 ($N=14$). Each node shows an agent that can either be an energy seller with photovoltaic (PV) generation or an energy buyer with no generation. Let us denote the number of buyer agents as N_B and the number of seller agents as N_S where $N = N_B + N_S$. Each buyers is indexed as i where $i \in \{1, 2, \dots, N_B\}$ and each sellers is indexed as j where $j \in \{1, 2, \dots, N_S\}$. In an outage situation of arbitrary duration, each i^{th} buyer is assumed to demand an energy amount $d_{i,j} \geq 0$ from the j^{th} seller. Depending on the network topology and considering a distance factor between the i^{th} buyer and j^{th} seller, each buyer can request different demands from every seller. We define the $1 \times N_S$ demand vector of the i^{th} buyer as \mathbf{d}_i . We also define the utility of i^{th} buyer as $U_i(\mathbf{d}_i)$ considering demand vector \mathbf{d}_i . We assume the utility function $U_i(\cdot)$ to be positive, increasing and strictly concave. Each buyer i is assumed to have a maximum demand D_i where $\sum_{j=1}^{N_S} d_{i,j} = \mathbf{d}_i^T \mathbf{1} \leq D_i$.

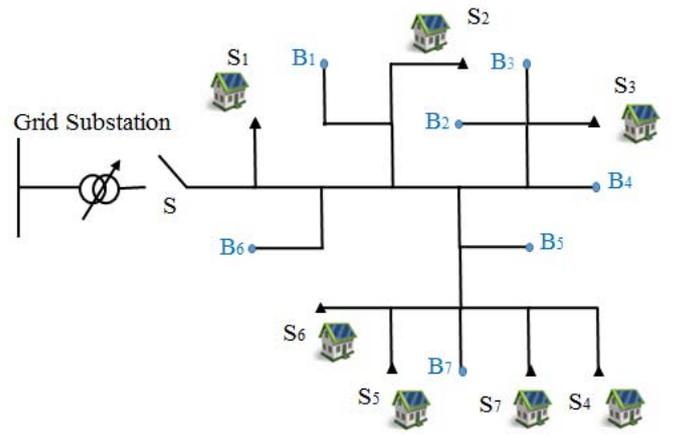


Fig. 1. Network topology for 7 buyers and 7 sellers (7×7) Case

As a node in the microgrid, the information of the network topology is known to the buyer, hence it will demand in such a way to minimize the transmission loss. For example, a buyer which is located near to a seller, will evidently demand all or most of its energy from that particular seller than those located farther, which makes it seller-specific. The transmission loss between two agents is assumed as the product of a distance factor, which incorporates the impedance of the connecting link, and the energy transferred between them [16]. Note that, considering such a transmission loss function will reflect the fact that the loss due to transmission of energy will proportionally depend upon both the amount of energy transmitted and the link distance between the agents.

On the other hand, consider each j^{th} seller has supply availability for the i^{th} buyer as $s_{j,i} \geq 0$. The $1 \times N_B$ supply vector of the j^{th} seller is defined as \mathbf{s}_j . Since each seller is assumed to own PV panels, it will incur some cost for discharging their individual batteries associated with their PV panels to supply specific amounts of energy to certain buyers. We assume a cost function $C_j(\mathbf{s}_j)$ which reflects the drop in utility of j^{th} seller delivering energy vector \mathbf{s}_j , assumed to be a positive, increasing and strictly convex function of \mathbf{s}_j . A quadratic cost function is a reasonable assumption for the cost incurred due to discharging and maintenance related expenses [10]. Such cost function reflects the fact that the more energy the seller wishes to supply, the more cost it will incur due to decreased lifetime of the battery and increased maintenance. Nevertheless, our proposed model and formulation will work with any type of such cost functions. We assume each seller to have a maximum limit of supply, defined as S_j where $\sum_{i=1}^{N_B} s_{j,i} = \mathbf{s}_j^T \mathbf{1} \leq S_j$. It is also assumed that the maximum amounts of supply from all sellers will suffice the demand of the all buyers in the outage duration.

It is certain that a viable market solution, market equilibrium, will be attained when both demand and supply matches between a buyer and a seller, or $s_{j,i} = d_{i,j}$; for $i \in \{1, 2, \dots, N_B\}$, $j \in \{1, 2, \dots, N_S\}$.

To establish a real time energy market, there exists a central control unit or controller that can be the utility provider who owns the network or a third party entity, capable to communicate with any agent. This controller is assumed to be able to facilitate the energy trading between any buyer and any seller in the network.

B. Problem formulation

Due to conflicting objectives of buyers and sellers, i.e. buyers try to maximize their utilities whereas sellers try to minimize their incurred cost, the controller will allocate optimal distribution of energy to meet the demand of the buyers. From a social perspective, it is desirable when the trading maximizes the social welfare and henceforth attains effective market equilibrium [17]. From that vantage, the role of the controller is to solve the social welfare maximization problem (*Objective 1*) to allocate energy for each buyer and seller for trading. Mathematically speaking,

$$\max_{\mathbf{d}_i, \mathbf{s}_j} \left\{ \sum_{i=1}^{N_B} U_i(\mathbf{d}_i) - \sum_{j=1}^{N_S} C_j(\mathbf{s}_j) \right\} \quad (1)$$

Subject to:

$$\sum_{j=1}^{N_S} d_{i,j} = \mathbf{d}_i^T \mathbf{1} \leq D_i; \quad i \in \{1, 2, \dots, N_B\}. \quad (2)$$

$$\sum_{i=1}^{N_B} s_{j,i} = \mathbf{s}_j^T \mathbf{1} \leq S_j; \quad j \in \{1, 2, \dots, N_S\}. \quad (3)$$

$$s_{j,i} = d_{i,j}; \quad i \in \{1, 2, \dots, N_B\}, \\ j \in \{1, 2, \dots, N_S\}. \quad (4)$$

The objective function in (1) is strictly concave with compact, convex constraints, hence possesses a unique optimal solution which can be described using KKT conditions [18]. Relaxation of constraints yields the following Lagrangian L_1 ,

$$L_1(\mathbf{d}_i, \mathbf{s}_j, \beta_i, \sigma_j, \mu_{i,j}) = \sum_{i=1}^{N_B} U_i(\mathbf{d}_i) - \sum_{j=1}^{N_S} C_j(\mathbf{s}_j) \\ - \sum_{i=1}^{N_B} \beta_i (\mathbf{d}_i^T \mathbf{1} - D_i) - \sum_{j=1}^{N_S} \sigma_j (\mathbf{s}_j^T \mathbf{1} - S_j) + \sum_{i=1}^{N_B} \sum_{j=1}^{N_S} \mu_{i,j} (s_{j,i} - d_{i,j}) \quad (5)$$

From the stationary conditions, it can be seen that

$$\nabla_{\mathbf{d}_i} L_1(\mathbf{d}_i, \mathbf{s}_j, \beta_i, \sigma_j, \mu_{i,j}) = \nabla_{\mathbf{d}_i} U_i(\mathbf{d}_i) - \beta_i \mathbf{1} - \boldsymbol{\mu}_i = 0 \quad (6)$$

$$\text{and } \nabla_{\mathbf{s}_j} L_1(\mathbf{d}_i, \mathbf{s}_j, \beta_i, \sigma_j, \mu_{i,j}) = \nabla_{\mathbf{s}_j} C_j(\mathbf{s}_j) + \sigma_j \mathbf{1} - \boldsymbol{\mu}_j = 0 \quad (7)$$

where $\beta_i \geq 0, \sigma_j \geq 0, \mu_{i,j}$ are Lagrange multipliers for the inequality and equality constraints (2), (3), (4). Complete information of all agents' utility and cost functions are required to enable the controller obtain the optimal solution using (6) and (7). Due to limitation of the complete information of agents, the controller needs to design a mechanism to extract the hidden information from the agents. Proposing an efficient (maximizes social welfare), individually rational (bidders will bid truthfully according to their private information), weakly budget balanced (controller never loses money for conducting the mechanism) double auction, the elicitation of hidden information can be done in a real, perfect, and competitive market with large number of participants having limited computational capabilities. The assumption that each agent is a price taking entity makes the energy market competitive as each agent will try to maximize their earning along with the maximization of their respective utilities.

III. AUCTION MECHANISM

In this section we present the concept of iterative double auction (IDA) used to elicit hidden information of agents to the controller. Assuming that a reliable communication link that facilitates flow of information between agents and the

controller exists, controller will perform an IDA that can meet the desirable economic property i.e. social welfare maximization. Fig. 2 shows the information flow between the controller and the agents. Each i^{th} buyer will submit a bid price $cb_{i,j} \geq 0$ for each j^{th} seller i.e. $1 \times N_S$ bid vector $\mathbf{c}\mathbf{b}_i$ and each j^{th} seller will also submit a bid $cs_{j,i} \geq 0$ for each i^{th} buyer i.e. $1 \times N_B$ bid vector $\mathbf{c}\mathbf{s}_j$, to the controller. These bids will reflect demand and supply of buyers and sellers respectively along with their preferences. After submission of bids, the controller will solve a different optimization problem, other than *Objective 1*, called Controller Allocation Problem (*Objective 2*) based on the submitted bids to allocate energy for every buyer and seller in order to achieve effective market equilibrium.

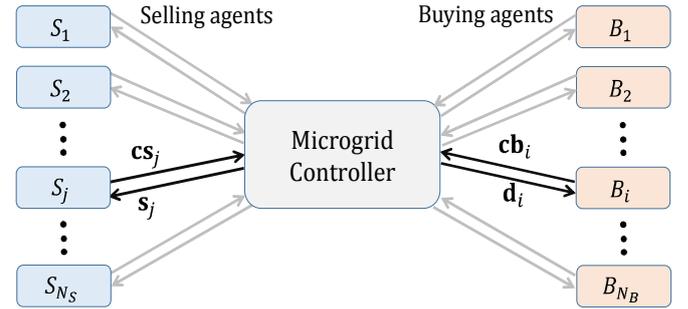


Fig. 2. Information flow in the energy network for IDA.

Solving *Objective 2* will result new optimal allocations \mathbf{d}_i and \mathbf{s}_j for announcement to the agents for trading. It would have been possible to attain the effective market equilibrium by performing the auction for once if all agents have the complete network information, which is not the case. This obliges performing the double auction mechanism for multiple iterations where each agent will solve its own utility maximization problem, namely Buyers' Optimization Problem (BOP) and Sellers' Optimization Problem (SOP) at each iteration to update their bid vectors according to the newly allocated demand and supply by the controller. As agents are selfish and non-cooperative, they are not concerned about the social objectives and try to maximize their own profit. Therefore, an additional role of controller is also to design some pricing rules i.e. settlement and earning rules for buyers and sellers respectively. Therefore, *Objective 2* can be formulated using the following objective function inspired by [13] with additional convex cost component as a positive and increasing quadratic function. Hence, *Objective 2* can be formulated as

$$\max_{\mathbf{d}_i, \mathbf{s}_j} \left\{ \sum_{i=1}^{N_B} \sum_{j=1}^{N_S} cb_{i,j} \log d_{i,j} - \frac{1}{2} cs_{j,i} s_{j,i}^2 \right\} \quad (8)$$

Subject to:

$$\sum_{j=1}^{N_S} d_{i,j} = \mathbf{d}_i^T \mathbf{1} \leq D_i; \quad i \in \{1, 2, \dots, N_B\}. \quad (9)$$

$$\sum_{i=1}^{N_B} s_{j,i} = \mathbf{s}_j^T \mathbf{1} \leq S_j; \quad j \in \{1, 2, \dots, N_S\}. \quad (10)$$

$$s_{j,i} = d_{i,j}; \quad i \in \{1,2, \dots, N_B\}, \\ j \in \{1,2, \dots, N_S\}. \quad (11)$$

The *Objective 2* has the same constraint set as the *Objective 1*, yet with a different strictly concave objective function which ensures existence of a unique optimal solution. The constraint relaxation of *Objective 2* by Lagrangian L_2 generates:

$$L_2(\mathbf{d}_i, \mathbf{s}_j, \beta_i, \sigma_j, \mu_{i,j}) = \sum_{i=1}^{N_B} \sum_{j=1}^{N_S} \left(cb_{i,j} \log d_{i,j} - \frac{1}{2} cs_{j,i} s_{j,i}^2 \right) - \\ \sum_{i=1}^{N_B} \beta_i (\mathbf{d}_i^T \mathbf{1} - D_i) - \sum_{j=1}^{N_S} \sigma_j (\mathbf{s}_j^T \mathbf{1} - S_j) + \sum_{i=1}^{N_B} \sum_{j=1}^{N_S} \mu_{i,j} (s_{j,i} - d_{i,j}) \quad (12)$$

Applying the stationary conditions yield

$$\nabla_{\mathbf{d}_i} L_2(\mathbf{d}_i, \mathbf{s}_j, \beta_i, \sigma_j, \mu_{i,j}) = \left[\frac{cb_{i,j}}{d_{i,j}} \right] - \beta_i \mathbf{1} - \boldsymbol{\mu}_i = 0 \quad (13)$$

$$\text{and } \nabla_{\mathbf{s}_j} L_2(\mathbf{d}_i, \mathbf{s}_j, \beta_i, \sigma_j, \mu_{i,j}) = [cs_{j,i} s_{j,i}] + \sigma_j \mathbf{1} - \boldsymbol{\mu}_j = 0 \quad (14)$$

which lead to the optimal allocation

$$d_{i,j} = \frac{cb_{i,j}}{\mu_{i,j} + \beta_i} \quad (15a) \quad \text{and} \quad s_{j,i} = \frac{\mu_{j,i} - \sigma_j}{cs_{j,i}}. \quad (15b)$$

Using (15a) and (15b), the controller will allocate optimal energy vectors between each buyer and seller.

To ensure that these optimal solutions will also solve the *Objective 1*, all KKT conditions along with the stationary conditions need to be matched for both *Objective 2* and *Objective 1*. The comparison results,

$$\nabla_{\mathbf{d}_i} U_i(\mathbf{d}_i) = \left[\frac{cb_{i,j}}{d_{i,j}} \right] \Rightarrow cb_{i,j} = d_{i,j} \cdot \frac{\partial}{\partial d_{i,j}} U_i(\mathbf{d}_i) \quad (16)$$

$$\nabla_{\mathbf{s}_j} C_j(\mathbf{s}_j) = [cs_{j,i} s_{j,i}] \Rightarrow cs_{j,i} = \frac{1}{s_{j,i}} \cdot \frac{\partial}{\partial s_{j,i}} C_j(\mathbf{s}_j). \quad (17)$$

This means, if each i^{th} buyer and each j^{th} seller submit their bids according to (16) and (17) respectively, then for controller solving *Objective 2* will be equivalent to solving *Objective 1*, which will elicit the hidden information from the market. So, the controller has to design settlement rule for buyers and earning rule for sellers in such a manner that each agents will be induced to submit their bids according to (16) and (17). Note that, all other KKT conditions except the stationary conditions are same for both L_1 and L_2 due to the same constraint set.

Now we define settlement rule for i^{th} buyer as $P_i(\mathbf{cb}_i)$ which depends on the allocated demand vector \mathbf{d}_i of that buyer. Similarly, an earning rule for j^{th} seller is defined as $E_j(\mathbf{cs}_j)$ which depends on the allocated supply vector \mathbf{s}_j for that seller. Note that, allocated demand and supply vectors are dependent upon buyers' and sellers' respective bids, which is evident from the *Objective 2* and optimal allocation rule (15a) and (15b). Since each agent is a non-cooperative and price-taking entity, it will try to maximize its overall utility which now depends upon its settlement or earning rule.

Thus, each i^{th} buyer will solve its own BOP to obtain its optimal bid vector \mathbf{cb}_i , or

$$\max_{\mathbf{cb}_i} \{U_i(\mathbf{d}_i) - P_i(\mathbf{cb}_i)\} \quad (18)$$

Similarly, each j^{th} seller will solve its own SOP in order to obtain its optimal bid vector \mathbf{cs}_j , or

$$\max_{\mathbf{cs}_j} \{E_j(\mathbf{cs}_j) - C_j(\mathbf{s}_j)\}. \quad (19)$$

The bids based on (16) and (17) will hold if the settlement and earning rules are chosen as follows:

$$P_i(\mathbf{cb}_i) = \sum_{j=1}^{N_S} cb_{i,j} \quad (20)$$

$$E_j(\mathbf{cs}_j) = \sum_{i=1}^{N_B} \frac{1}{cs_{j,i}} (\sigma_j - \mu_{i,j})^2. \quad (21)$$

Pricing rules in (20) and (21) are constructed in such a way that the optimal bids of agents will aid the controller to obtain the socially optimal solution which is nothing but the optimal solution of *Objective 1*. Note that in (20) i^{th} buyer will exactly pay the bid $cb_{i,j}$ that it has submitted to the controller for the j^{th} seller. However for the j^{th} seller, by (21) its earning is inversely proportional to their respective bids $cs_{j,i}$, which implies that a seller is less paid if it submits high offers to sell. This will be further clarified in the simulation results in section V.

IV. IDA ALGORITHM

In this section we present an IDA algorithm, as shown below, based on the pricing rules we obtained in (20) and (21).

Algorithm

1. Agents place initial bid vectors $\mathbf{cb}^{(0)}$, $\mathbf{cs}^{(0)}$, $t \leftarrow 0$.
2. Agents submit demand and supply limits as follows.
 - Buyers: D_i .
 - Sellers: S_j .
3. Repeat until converged
 - $t \leftarrow t + 1$
 - Convergence criteria: $|cb^{(t)}_{i,j} - cb^{(t-1)}_{i,j}| < \varepsilon$,
 $|cs^{(t)}_{j,i} - cs^{(t-1)}_{j,i}| < \varepsilon$.
 - a. Obtain $d_i, s_j, \beta_i, \sigma_j, \mu_{i,j}$ by solving *Objective 2*.
 - b. Compute $P_i(\mathbf{cb}_i)$ and $E_j(\mathbf{cs}_j)$ using (20) and (21).
 - c. Announce $\mathbf{d}_i, \mathbf{s}_j$; the optimal solutions of *Objective 2*.
 - d. Agents compute bid prices:
 - Buyers solve BOP. Update bid vectors $\mathbf{cb}^{(t)}$ using (16).
 - Sellers solve SOP. Update bid vectors $\mathbf{cs}^{(t)}$ using (17).
 - e. Agents submit bid vectors $\mathbf{cb}^{(t)}$, $\mathbf{cs}^{(t)}$.

End

In the first iteration, each buyer will submit their initial bid vector for all sellers to the controller. On the other side, each seller will submit their bid vector for all buyers. Note that, each buyer will submit their initial bids based on the distance factor, with specific seller preferences. Each buyer-seller will also specify their maximum demand-supply limits respectively. Using this initial information, controller will solve *Objective 2* in order to allocate the demand and supply based on their individual bids. In turn, the controller will announce these new allocations to the buyers and sellers. The controller will also compute the settlements and earnings for each buyer and seller. Based on these new allocations, buyers solve total N_B number of BOPs and sellers solve total N_S number of SOPs to obtain their optimal bids for the next iteration. These new bids will be submitted to the controller by each agent. Controller will check the termination condition. The IDA algorithm will terminate if updated bids are equal to the previous iteration's bids. New

settlements and earnings as well as the latest allocation of energy will be determined by the controller.

Using (16) and (17), each agent will bid truthfully and maximize its own utility. Additionally, it can be shown that auctioneer will not lose anything while conducting the IDA i.e. the auction is weakly budget balanced. Furthermore, every agent's utility will be at least non-negative as their utility functions are assumed such [21, 14]. Hence in such a setting, a truthful, individually rational, weakly budget balanced IDA mechanism can attain a socially optimal energy allocation between buyers and sellers during an outage with incomplete information to the controller. The convergence of the proposed IDA algorithm is shown in the following simulations results section V.

V. SIMULATION RESULTS

A. Simulation setup

Considering the model of Fig. 1 as a microgrid network of 14 agents where the buyers are denoted as $B_i, i \in \{1,2, \dots, 7\}$ and sellers are denoted as $S_j, j \in \{1,2, \dots, 7\}$. Each connecting transmission section is assumed as one unit of distance with a constant impedance. We assumed the distance factor between i^{th} buyer and j^{th} seller as $z_{i,j}$, which is directly proportional to number connecting units present between them multiplied by an adjusting factor of 0.01 to convert it to a reasonable range. For example, the distance factor between buyer 1 and sellers 1 and 2 are $0.01 * 6$ and $0.01 * 4$ units respectively. See Table I for a complete list of the distance factor. This particular distance factor makes all buyers seller-specific. The utility of i^{th} buyer is designed as a positive, increasing and concave function, $U_i(\mathbf{d}_i) = b * \sum_{j=1}^7 \log(d_{i,j} - d_{i,j} * z_{i,j} + 1)$ where b is a constant utility factor and the term $d_{i,j} * z_{i,j}$ incorporates the transmission loss. On the other hand, the cost function of j^{th} seller is modeled as $C_j(\mathbf{s}_j) = a_{1j}(\sum_{i=1}^7 s_{j,i}^2) + a_{2j}(s_{j,i})$ where (a_{1j}, a_{2j}) are cost factor constants. It is assumed that every seller has a different positive, increasing, and convex cost function depending upon its individual cost factor (a_{1j}, a_{2j}) .

B. Simulation results

Three cases with 5 buyers 5 sellers (5×5), 7 buyers 7 sellers (7×7), and 10 buyers 10 sellers (10×10) has been simulated and reported here. Major part of the simulation result and analysis is based on the 7×7 case and can be extended to any number of sellers and buyers.

Fig. 3(a), 3(b), and 3(c) show the convergence of social welfare function for 5×5, 7×7, and 10×10 cases respectively using the proposed algorithm. Fig. 3(d) shows the convergence plot of IDA mechanism for the 7×7 case where the sum of differences of all agents' previous and updated bids has been plotted with respect to number of iterations. Note that, the proposed algorithm is very fast and practical. It takes about 8.30, 41.06, 117.00 seconds for the 5×5, 7×7, 10×10 cases respectively to converge using MATLAB's built in optimization functions in a 64GB RAM, Intel Core i7, 3.70Ghz computer.

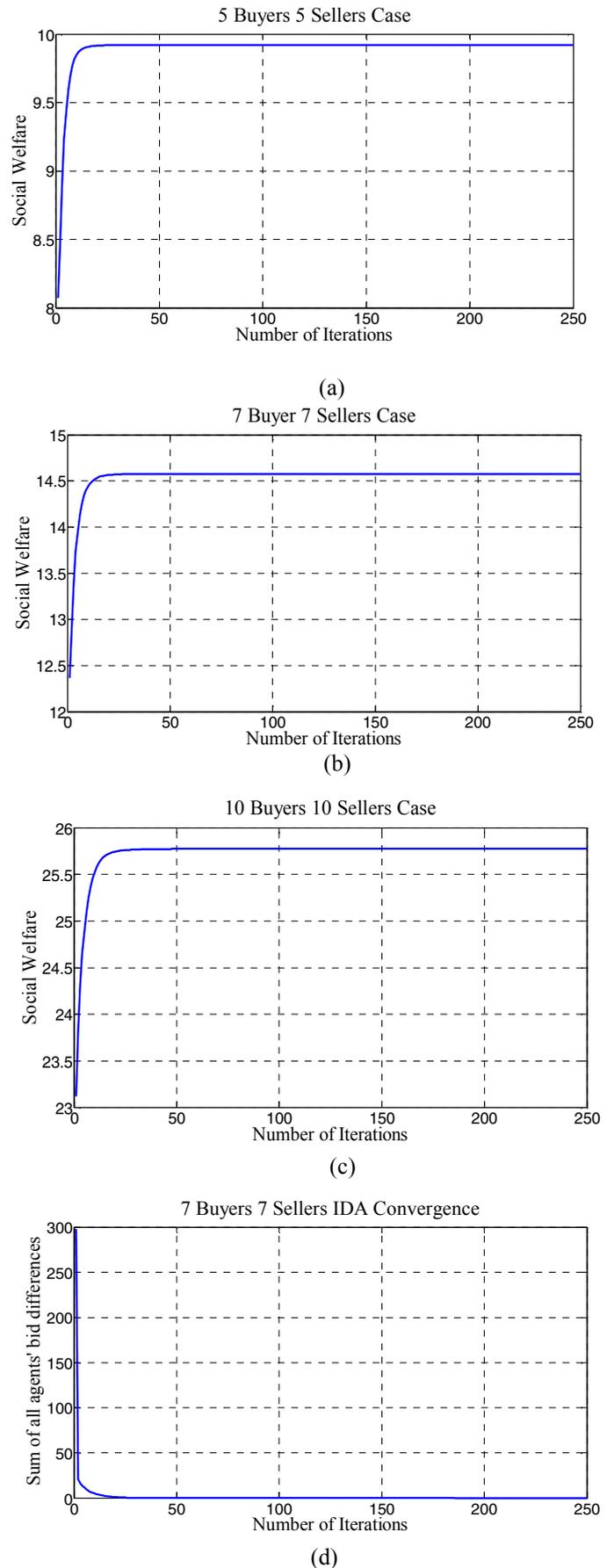


Fig.3. Social Welfare w.r.t. no of iterations in a) 5×5 case b) 7×7 case c) 10×10 case and d) Sum of differences of all agents' previous and updated bids w.r.t. no of iterations in 7×7 case.

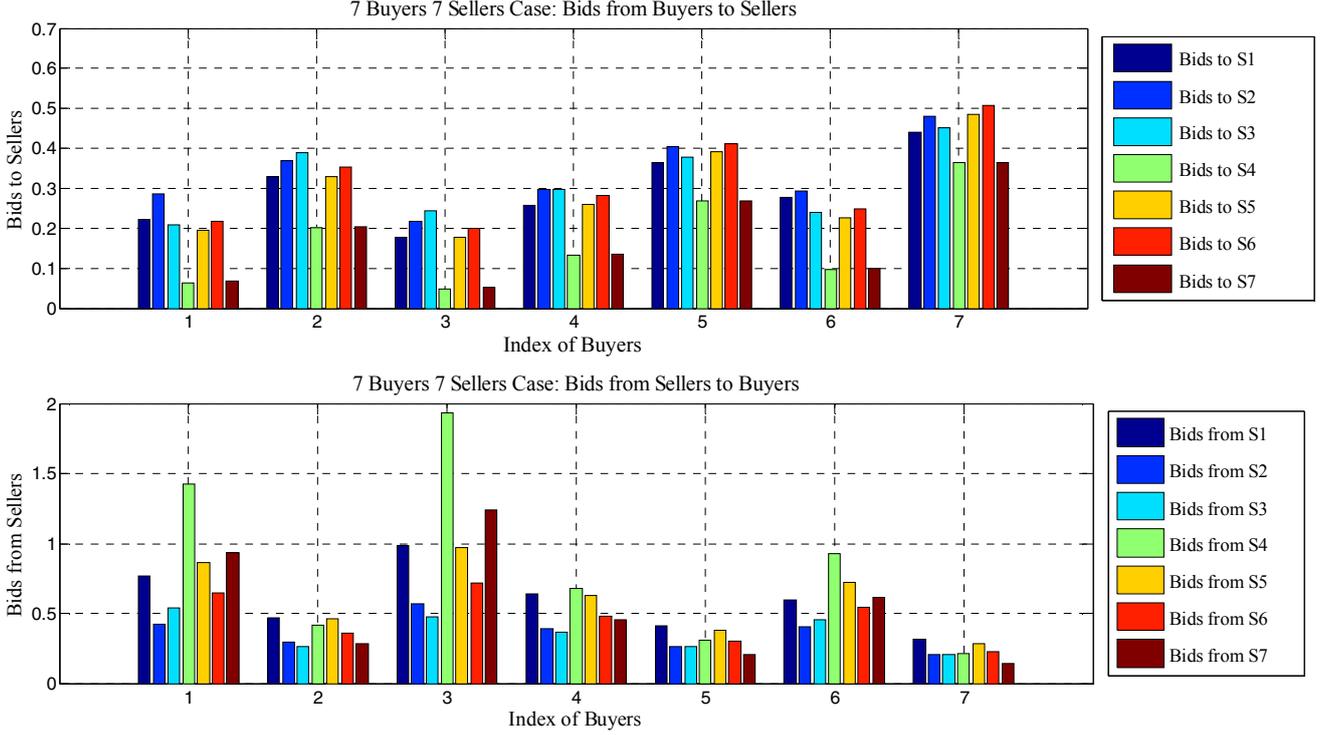


Fig. 4. Bids from buyers to sellers ($cb_{i,j}$) and bids from sellers to buyers ($cs_{j,i}$) for 7×7 case

Fig. 4 shows the individual bids of agents for a 7×7 case with horizontal axis showing the buyers' index for both plots. Note that, the 7 colored bars for buyer B_1 shows the amounts of its bids to the 7 sellers in the upper plot where the 7 colored bars in the lower plot shows the amount of bids it receives from the sellers. Notice that if a certain seller places high bid for a certain buyer, then that specific buyer will decide to bid less for that seller. For example, bids from seller S_4 to buyer B_3 is highest compared to the bids from other sellers to buyer B_3 . Thus, buyer B_3 will submit minimum bid to seller S_4 . This analysis holds for all other agents' bids.

Table I shows the assumed distance factors between agents which accounts for inter-agent distances and impedances. Considering Fig. 1 and Table I, as the distance for B_3 to S_4 is long compared to other sellers, i.e. $z_{3,4} = 0.08$, B_3 will want to buy least amount of energy from S_4 , i.e. $d_{3,4} = s_{4,3} = 0.0365$, to minimize its transmission loss. In addition to that, bid vectors for buyer B_3 and seller S_4 , i.e. $\mathbf{cb}_3 = [0.176, 0.216, 0.244, 0.049, 0.178, 0.200, 0.52]$ and $\mathbf{cs}_4 = [1.426, 0.415, 1.930, 0.677, 0.308, 0.926, 0.210]$, show that the bid of buyer B_3 to seller S_4 is minimum, submitted based on its distance factor between them, whereas seller S_4 's bid is highest to buyer B_3 . Thus, the energy traded between them is minimal. The energy demand–supply for the 7×7 case is shown in Table II which reflects the analysis above for all agents.

TABLE I: Distance factors $z_{i,j}$: Row denotes i^{th} buyer. Column denotes j^{th} seller.

$z_{i,j}$	S_1	S_2	S_3	S_4	S_5	S_6	S_7
B_1	0.06	0.04	0.07	0.09	0.08	0.08	0.08
B_2	0.07	0.07	0.02	0.08	0.07	0.07	0.07
B_3	0.07	0.07	0.02	0.08	0.07	0.07	0.07
B_4	0.06	0.06	0.03	0.07	0.06	0.06	0.06
B_5	0.06	0.06	0.05	0.05	0.04	0.04	0.04
B_6	0.04	0.06	0.07	0.09	0.08	0.08	0.08
B_7	0.07	0.07	0.06	0.04	0.03	0.03	0.03

TABLE II: Demands $d_{i,j}$: Row denotes i^{th} buyer. Column denotes j^{th} seller.

$d_{i,j}$	S_1	S_2	S_3	S_4	S_5	S_6	S_7
B_1	0.1839	0.2461	0.1729	0.0495	0.1620	0.1843	0.0513
B_2	0.3016	0.3504	0.3570	0.1699	0.3031	0.3302	0.1694
B_3	0.1431	0.1812	0.1984	0.0365	0.1443	0.1655	0.0385
B_4	0.2207	0.2632	0.2541	0.1042	0.2221	0.2456	0.1050
B_5	0.3427	0.3938	0.3545	0.2289	0.3665	0.3948	0.2273
B_6	0.2376	0.2575	0.2044	0.0761	0.1936	0.2170	0.0774
B_7	0.4466	0.5066	0.4587	0.3350	0.4929	0.5261	0.3312

VI. CONCLUSION AND FUTURE WORK

As seen in the earlier sections, the proposed iterative double auction is (i) efficient, i.e. maximizes the social welfare of the agents, (ii) does not require full information of the agents, (iii) realizable in an entirely distributed manner, (iv) considers asymmetric preferences of the participating agents, and as the simulation results suggest, (v) effective both in terms of computation speed as well as in maximizing the social welfare. Furthermore, although not addressed here, the approach is also (vi) truthful, and (vii) strategy proof [21, 15].

A broad framework that models microgrids with smart homes represented as agents, and with various kinds of loads that are distributed across multiple time frames, has recently been proposed [20]. The suitability of the IDA described here, with further enhancements, is under active consideration by the authors of this paper. Moreover, the use of share auctions [19] that can more directly minimize transmission line losses is under investigation.

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